Photogrammetric Analysis and 3D Modeling of Early Muslim Glazed Pottery Collection at Hazara University Museum, Pakistan

Shakirullah / Muhammad Zahoor / Asim Rasheed

Abstract

Today, photogrammetric and 3D computer modeling is the essential and fundamental techniques utilized in archaeology and other heritage-related fields. In the areas of conservation, preservation, restoration, and mediation of architectural, archaeological, and cultural heritage, they offer essential responses to scientific needs. There aren't many computer-based automation tools for pottery classification in the realm of archaeology. Lacking computer vision-assisted technologies, archaeologists are unable to see a full pot from a single shattered fragment in three dimensions. The strength and widespread use of computer-based automation techniques have not yet had a truly positive impact on Pakistan's archaeology. The methods for obtaining 3D data for fragments have been extensively addressed, but the methods for virtual reconstruction have not received enough attention. There is a large collection of early Muslim glazed pottery housed in Hazara University Museum of Archaeology. The aim of the present study was to reconstruct and complete the object from pottery fragments and to determine the drawing, designs, and color combination of selected pottery. For this purpose, fragments of two different pots from the same collection were selected. The result of this study revealed a complete 3D model of each broken glazed ceramic pot with help of proposed techniques and computer applications.

Keywords: Photogrammetry, Muslim, Glazed, Pottery, 3D reconstruction.

1. Introduction

The Ghaznavid are credited with the invention of glazed pottery. Due of its special qualities, glazed ceramics are well-known and are displayed in numerous museums across the world (Gulmini et al., 2013). The Hazara archaeological and Ethnological Museum, Hazara University Mansehra has a substantial collection of glazed ceramics from the early Muslim era. This collection consists of many kinds of bowls, plates, and lamps with calligraphy, floral and geometric designs, birds, and human figures as

ornamental themes. The shards, which were formerly in a private collection, were presented by the former Governor, Khyber Pakhtunkhwa Province (formerly known as NWFP). Through scientific analysis of this unique collection, can place it in the Ghaznavid period originated at southern Afghanistan's in two famous schools of glazed ceramics i.e. Lashkari and old Bust Bazars.

Archaeologist Jean-Claude Gardin (1977) conducted a thorough investigation of the pottery found in Bust and Lashkar-i Bazar and divided all of the discoveries into many groups. A first subdivision dealt with dividing pottery into glazed and unglazed categories. Unglazed pottery was further separated into five groups and subsequently into series depending on form and decoration, forming a relatively consistent collection as far as paste features are concerned. In terms of glazed pottery, Gardin divided it into three main categories that were also based on chronology. These categories were distinguished by painted decoration on engobe, painted and incised decoration on engobe, and painted decoration without engobe. The 3D reconstruction of ancient pottery has been done in the rest of the world while in Pakistan rare attempts have been made so far.

2. Review of Literature

In the middle of the 1980s, researchers were considering methods that would sketch, preserve, store, and restore the excavated fragments. Many strategies are used to put together jigsaw puzzles and reconstitute their 3-D models in neuroscientific matching items. The process of using B-splines for shape illustration is described by Half and Laflin (1984). It enables three-dimensional curved solids of rotation to build figures based on the B-spline profile, producing sketches of the ceramics. This technique has the advantage of allowing photographs of the 3D construction of pottery profiles, compression prior to computer storage area for future display, and record examination.

An accurate model for the estimate of the pottery profile was provided by Hal and Flusser in (1997). By determining the radius of one sherd, they were able to establish the parameters defining the subject's diameter and edges. They also provide proof that the crossings of the fragment's area with a number of parallel planes result in parallel arcs with a single axis and varying widths. The piece is right-oriented for making a circular arc. Essentially, the estimated laser plane and the fragment surface intersect to create the circular arc. Both synthetic and actual ceramic data were used for testing. According to the bottom-up design, Sablatnigand and Menard (1997) employed the profile curvity for the reconstruction full pot and its computerized classification. Regardless of the absence of other ceramic fragments, Üçolukand and Toroslu (1999) concentrated on damaged three-D surfaces reaping the corresponding of consecutive contour pixels of two pottery fragments. Using simulated broken objects, a Noise Tolerant set of criteria were used to achieve first-class matching. Through analysis of the damaged surface boundary curves, they fit and align the vase fragments.

Based on the vessel axis in general location, the profile-curve and zaxis, a portion of the exterior pottery, smash-curves to the complete pottery, and 3D euclidean variations, Willis and Andrews (2001) calculated the statistics of the pot herds, enabling the geometric parameters to merge and form, as a result. The framework for the automated reconstruction of broken pottery was developed by Andrews and Laidlaw in (2002). The geometric correlations were constructed using X2 statistics.

Papaioannou, Karabassi, and Theoharis (2002) focused on the matching and alignment of parts for an automated 3D reconstruction. Through computing, the algebraic surface with axial symmetry was used. Kampel and Sablatnig (2003) based their automated reconstruction on the profile of the fragments. An automatic archival was created by Kampel and Sablatnig (2003b) to classify and recreate the ceramics. A semi-automatic device was invented by Melero, Torres, and León (2003). Pots were mechanically put together by Willis, Orriols and Cooper (2004) using three-dimensional pieces.

The fundamental concept that sets their methodology apart is how they present the examination of two assemblages of Israeli Iron Age ceramics. By accessing data through restrictions, Lui and Pottmann (2006) examined 3D reconstruction. These restrictions may specify the surface type or establish geometric relationships between the surface of the object's component pieces, such as orthogonality, parallelity, and others. Instead of mechanically reassembling 3D solid objects, Huang et al. (2006) developed a segmentation technique that is entirely based on the graph cuts method based on region functions.

A typical vessel model was created utilising 3D convexhull technology by Cohen, Zhang, and Jeppson (2010). The Graphical Processing Unit (GPU) depth Maps approach was recommended by Belenguer and Vidal (2012). Kashihara (2012) suggested a silhouettesbased technique for automatic reconstruction. Goel and Singh (2005) described the features and internal workings of a computer-aided design tool used to classify and recreate ceramic artefacts in archaeology. With limited resources, Barreau and Nicolas (2014) used photogrammetry to recreate pottery from ceramic shards. Their research provides a clear explanation of their methodology. Kampel and Sablatn (2004) used automatic fragment orientation to focus on the parentage of the profile line.

Rotation axis and profile were employed to restore the ceramics in which the laser scanner and light scanner were used. A technique for creating 3D models of ancient Greek pottery is presented by Koutsoudis et al., (2009). From these models, digital signatures were recovered using algorithms. The software automatically or partially classified that signature. The results of a comparison of human and digital reconstruction procedures in terms of robustness, integrity, and usefulness are presented by Eleni Kotoula (2016). Three distinct semi-automatic fragment alignment and matching algorithms were used. It is advised to employ a combination of strategies when using various software applications. For the modelling methodologies, the applications of the digitally restored artefact were discussed.

3. Methodology

A systematic research methodology was used in this study. Due to the scarcity of primary data sources, initial secondary data collection took the form of papers on 3D reconstruction. This study can be used as a reference for majority of investigations that have been done on the 3D reconstruction of pottery. The fundamental information, including the texture, structural types, and pot measurements, were provided by the Hazara University Museum. The reconstruction and examination of ceramics were aided by these dimensions and textures. In the theoretical context, the technique is applied to literary and archaeological sources. The common practices of searching libraries and the internet for literature about the development of 3D ceramic reconstruction also yields literary sources.

Acquiring Data

In order to take specific measurements for analysis, a thorough study of 3D reconstruction of pottery was conducted. This was followed by careful photography of the pot in various poses to obtain the texture for the 3D reconstruction.

Photography

The study relies on photogrammetric reconstruction of archaeological ceramics, hence a high-end digital single-lens reflex camera with a good lens and good resolution is required. Better outcomes and high-quality images will emerge from this. Without photos, archaeological research is insufficient. Every pot fragment was photographed in detail, with thorough photography being used. From these pictures, the Left Texture aspects from the field notes were retrieved. The future conservation, preservation, and restoration efforts will benefit from all these field photos, which are also a type of historical record.

Adobe Photoshop

Adobe Photoshop is a fantastic tool for producing, altering, managing, and maintaining images for the online and printed media. It might very well be used to create web designs for a website or to update high-resolution photos for crucial introductions. With "Edit" for object type, "layer" for creating distinct layers, and "Filter" for geometric precision, the creation of geometries and textures was processed in Adobe Photoshop. Adobe Photoshop created the texture from number of extracted image fragments and applied it to the mesh's surface. After that, the generated mesh was exported in ".obj" format and loaded into Maya for modelling.

Autodesk Maya

Maya was employed for the ceramic modelling. Maya, sometimes known as Autodesk Maya or simply Maya, is a 3D computer graphics program. A 3D modelling and animation program that supports 3D printing is Autodesk Maya. A program called Maya is used to create 3D objects for use in architecture, television, and film. In Maya, there are four different layouts. The left views show the object from the top, front, and side, while the perspective view is in the top right corner. Afterward, choose the edge and extrude the shape in accordance with the reference image. Now, there are several choices for editing the mashup. Numerous more terms include fractional, absolute, offset space, global, and local. Choose a mesh object, then choose the faces and edges you want to bevel. The chosen edges and faces will then bevel in Maya. When the model is finished, the UV unwrapping process comes next. After unwrapping, take a texture snapshot,

which you may then import into Photoshop to edit. and after that, apply the texture to the model in Maya.

Matlab

To analyse ceramics, Matlab was employed. Matlab combines a programming language that natively expresses matrix and array mathematics with a desktop environment optimised for iterative analysis and design processes. For writing scripts that mix code, output, and formatted text in an executable notebook, it comes with a live editor.

4. Analysis and procedure

The applications are mostly used by archaeologists for ceramic analysis. First, choose a piece of the fragment preserved in the 2D profile for input. The GUI was displayed on a screen when the Matlab application is first executed. The file selection window thereafter opened as we perused the input and the selected ceramic fragment's 2D profile (which is stored as a.png file) was selected. We Set the radius to mm and then press the Build button to create a 3D model of the original body's selected fragment. Also here we will get the analysis of this pottery on the basis of radius, tangent, curvature & capacity of the pottery.

Radius

r=radius (the line segment from center to parameter, dr= the change of radius anlong the y-axix of rotation. Mathematically to calculate a curvature at a particular point following formula is used.

Radius of curvature=
$$\frac{[1+(\frac{dy}{dx})^2]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left(\frac{y^2 + x^2}{y^3}\right)$$

That is $\frac{d^2y}{dx^2}$. Second order differential equation.

Tangent

(T) tangent: A line that touches any straightforward point on a curve. Dt is the difference in the curve's tangent. The result of ds from the reference point is dt as shown in figure 1.



Fig. 1: a) Radius of circle, b) Tangent of circle.

Curvature

The pace in which a curve's direction changes in relation to its length is known as "curvature" in mathematics. Dc=change of curvature is the function of dt with ds.

$$k = \left| \frac{dt}{ds} \right|$$

Capacity

Capacity is shown in \boldsymbol{v} mathematically volume of vessel/Pottery is calculated by following.

 $V = r^2 \pi H i.e \pi = 3.14 \text{ or } 22/7.$

5. Results

Sample-1 (Bowl: Acc #: HUM 00368)





Fig. 2: Different steps during Computer programming (Bowl: Acc #: HUM 00368).

Fig. 3: Output result after processing different steps (Bowl: Acc. #: HUM 00368).

Vol. 45 No. 1, June 2022

Sample-2 (Bowl: Acc. #: HUM 00392)

Step-1 Begin initialization code

ggui_Singleton = 2; gui_State = struct('gui_Name', bowl 2, ... 'gui_Singleton', gui_Singleton, ... 'gui_OpeningFcn', @Pottery_cluster_OpeningFcn, ... 'gui_OutputFcn', @Pottery_cluster_OutputFcn, ... 'gui_LayoutFcn', [], ... 'gui_Callback', []);

Step-2 Draw boundary from fragment from an input for Hdx-2: -2: 2

$$\begin{split} B=& \text{imread}([PathName FileName]); \\ B &= B(: , , 2); [r, c] &= \text{find}(B, 2, \text{'first'}); \\ B &= bwtraceboundary(B, [r, c], N'); \\ tmp=B(: , 2); B(: , 2)=B(: , 4); B(: , 4)=tmp+max(tmp); \\ (-tmp]=min(B(: , 1)); tmp=B(tmp, :); \end{split}$$

Step-3 Smoothing the curve

$$\begin{split} & idx{=}1: 9: length(B); B{=}B(idx;;); \\ & axes(landles, axes); plot(B(: 2,)B(: ,4)), hold on \\ R{=}ta2doub[cg(eltandles, cdit4; String)); \\ & axis{=max(B(: ,1)){+}R; \\ & plot([axis axis], [min(B(: , 4)) max(B(: ,4))], '-k') \\ text(axis+5; max(B(: , 4)), 'Axis of rotation') \\ text(axis+5; max(B(: , 4)), 'Axis of rotation') \\ text(axis+5; max(B(: , 4)), 'Ai; of rotation') \\ xlim([min(B(: , 1)), (4*axis-min(B(: , 1)))]) \\ mirror{=}B(: ,2){+}^4(axis-B(: 2, 2);) \\ plot(mirro,B(: , 4), 'Y) \end{split}$$

Step-4 Radius

 $\begin{array}{l} [-,idx] = \max(B(:,4)); ref=B(dx,:); tmp=0; \\ hold captotjet([1),ref(4),"hold off \\ textref(1),1,1"et(4); Refrence point) \\ ra(idt) xxis:ref(2); \\ for i=idx+1: length(B) \\ s(0)=mp+sqt(B(i-2,2)-B(i,2))^3+((B(i-2,4)-B(i,4))^4)); \\ tmp=i0; \\ ra(i)=xxis:B(i,1); \\ end \\ tmp=0; \\ for i=idx-2: -2.2 \\ s(0)=mp-sqt((B(i+2,2)-B(i,2))^2+((B(i+4,4)-B(i,4))^2)); \\ tmp=i0; \\ ra(i)=xxis:B(i-4): \end{array}$

Step-5 Tangent

Step-6 Curvature

K=diff(heta)./(diff(s)); K[hengh(K)+1]=K(end); axes(handles axes1) pbb(s,na),itle(Radius),grid on axes(handles axes4) pbb(s,K),itle(Clarveture),grid on axes(handles axes5) pbb(s,K),the(Curvature),grid on

Step-7 Determining the pottery capacity

[~,idx1]=max(B(: ,4)); [~,idx4]=min(B(: ,4)); tmp=B(idx1,:); x1=tmp(2); y1=tmp(4); tmp=B(idx2,:); x2=tmp(2); y2=tmp(4); $m{=}(y1{-}y2){/}(x2{-}x1); \ y{=}@(x) \ m^{\bullet}(x{-}x2){+}y2;$ inner=[]; if m<0 for i=2: length(B) tmp=B(i,:); xt=tmp(2); yt=tmp(4); yr=y(xt); if yt>yr, inner=[inner;tmp]; end end elseif m>0 for i=2: length(B) tmp=B(i,:); xt=tmp(2); yt=tmp(4); yr=y(xt); if yt<yr, inner=[inner;tmp]; end end end [tmp,I]=sort(inner(: ,4),'descend'); tmp2=inner(: ,2); tmp2=tmp2(I); inner=[tmp4 tmp]; Vol=0: for i=2: length(inner)-4 tmp=inner(i,:); r=axis-inner(i,2); dy=inner(i,4)-inner(i+2,4); Vol=Vol+(pi*(r^2)*dy); end Vol=(1e-1)*Vol; title(['The capacity = ' num2str(Vol) ' L'])



Fig. 4: Different steps during Computer programming (Bowl: Acc #: HUM 00392).

Figure 5: Output result after processing different steps (Bowl: Acc. #: HUM 00392).

3D Reconstruction of Broken Pottery

Sample-1 Bowl: Acc. #: HUM 00368



Fig. 6: a) Original broken pot before process, b) 3D model, c) production of a complete pot after process.

Sample-2 (Bowl: Acc. #: HUM 00392)



Figure 7: a) Original broken pot before process, b) 3D model, c) production of a complete pot after process

6. Discussions

Archaeology and other heritage fields use photogrammetric and 3D computer modelling as essential and fundamental techniques nowadays. In the areas of conservation, preservation, restoration, and mediation of architectural, archaeological, and cultural heritage, they offer the essential responses to scientific needs. There aren't many computer-based automated tools for reconstructing pottery in the field of archaeology. Lack of support for computer vision-aided tools prevents archaeologists from being able to envision a full pot from a single shattered fragment in three dimensions. Although computer-based automation techniques are powerful and widely used, archaeology has not been completely impacted by them. Although the strategies for obtaining 3D data for fragments have been extensively addressed, not enough attention has been paid to them.

In order to share knowledge about cutting-edge techniques and methodologies utilised in the archaeological reconstruction of pottery, this research combines fundamental information on 3D reconstruction as a distinct field. Glazed ceramics from the early Muslim era are on display at the Hazara University Museum. There are many kinds of bowls, plates, lamps, calligraphic artwork, pheasants, and ducks, as well as floral and geometrical designs. With the exception of Pakistan, the rest of the globe has completed the 3D reconstruction of pottery. There are not many publications on 3D reconstruction, and each method addresses a distinct classification subproblem for a data set, such as ceramics, pottery, or mixed or textured pottery, for example. Furthermore, no existing technique makes use of the pottery's front and back view properties.

Here, we talked about a piece of software that can create full 3D pots. We demonstrated how to make a variety of arbitrary artificial 3D pots using some traditional Ghaznavid pottery forms. The ancient Ghaznavid pottery was highlighted because, of the entire artefact types that might have a significant impact on human history and living, ceramics are unquestionably the most significant. The idea of 3D pot parameterization for analysis has been our main focus. A programme was created to quickly offer a data set for the creation of 3D shapes centered on ceramic artefacts in terms of standard coordinate systems and data structures. The three primary parts of our suggested system are discussed in this section. The features were included in the reference dataset for the texture.

The first component was to extracts a collection of photographs, getting every viewpoint from every pot. The photogrammetry feature of

Adobe Photoshop was used for 3D restoration. This software was chosen for its user-friendliness and capacity to produce incredibly detailed 3D models. The goal is to capture all viewing angles. A first sequence of photographs illustrating a semicircle with a distance camera-object as regular as feasible will be taken in order to get the required outcomes. The object rotated through 180 degrees to finish the series and receive the initial covering. To capture the largest possible region, more photo collections were taken from various angles.

The second one was to generates a 3D representation of the pot into the Mava 3D model using geometric features. Photoshop was used to process the creation of the geometry and textures. The points cloud's colour information used to calculate the mesh's colour. With help of Photoshop texture from various image fragments was extracted which later applied to the mesh's surface. Both a "average" fusion mode and a "generic" mapping mode was available for this stage. The resulting mesh processed through scaling. A measure of distance was taken with the tool "rule" between two points whose real distance is known. The proportionality factor between these two distances determined by dividing the real distance by the measured distance on the mesh. Maya is made up of five modules: Reading, Math, Display, Reconstruction, and Output. Reading reads various types of digital models; Math contains a large mathematical library, particularly the matrix library and vector library; Display displays the model on the screen; Reconstruction restores the broken sherds using rotation axis and complete profile; and Output saves and outputs the restored model. The geometric features generate a radius, which yields the value of the tangent and carvature of the pot using Matlab for the study.

The third component was to extracts the diameter of each pottery. We can reuse sets of commands by storing them in program files using both scripts and functions. Since they save commands exactly as we would input them at the command line, scripts are the simplest kind of software. Functions offer more versatility because we can input values and receive output values in return.

7. Conclusions

Digital reconstruction is a very recent field. It would undoubtedly be advantageous to conduct research, build tools to make it accessible, and try to promote it among archaeologists. The study of ceramics has been regarded as a valuable cultural resource for reconstructing number of aspects of ancient society. Computer applications have so remained underdeveloped in Pakistan, especially regard to the digitization of artifacts in archaeology. In this study, pottery from the Hazara University Museum Mansehra was examined, and an application is shown that may be utilized to repair shattered pottery pieces and add missing textures. Although we can complete the entire profile (model, shape, and texture) of the pot, the suggested technique and computer program is not just for reconstructing shattered pottery but also for an excavated piece of pottery. This would help not just Pakistani archaeologists but also several other academics who are interested in ceramic reconstruction.

References

Andrews, S. and D.H. Laidlaw (2002) Toward a Framework for Assembling Broken Pottery Vessels, *Proceeding of 18th International Conference on Artificial Intelligence: American Association for Artificial Intelligence*, pp. 945 - 946.

Barreau, J.B. and T. Nicolas (2014) Ceramics Fragments Digitization by Photogrammetry, Reconstructions and Applications, *International Journal of Heritage in the Digital Era, EuroMed*, *3*(*4*), pp. 643 - 646.

Belenguer, C.S. and E.V. Vidal (2012) Archaeological Fragment Characterization and 3D Reconstruction based on Projective GPU Depth Maps, *IEEE 18th International Conference on VSMM*: 275 - 282.

Cohen, F., Z. Zhang and P. Jeppson (2010) Virtual reconstruction of archaeological vessels using convex hulls of surface markings, *IEEE Computer Society Conference on Computer Vision and Pattern Recognition Workshops (CVPRW)*, ISSN (2160-7508): 55–61.

Gardin, J.C. (1977) The Study of Central Asiatic Pottery: Some Reflections on Publication. *American Journal of Archaeology*, *81*(1): 80 - 81.

Goel, S. and P. Singh (2005) Computer Vision Aided Pottery Classification and Reconstruction, *INDO US Science and Technology Forumon Digital Archaeology*.

Gulmini, M., R. Giannini, A.M. Lega, G. Manna and P. Mirti (2013) A Technology of Production of Ghaznavid Glazed Pottery from Bust and

Lashkari Bazaar (Afghanistan), Archaeomatery 55, (4): 569 - 590.

Halíř, R. and J. Flusser, (1997) Estimation of Profiles of Sherds of Archaeological Pottery, *Proceeding of the Czech Pattern Recognition Workshop '97. Proceedings, p. 126-130, T. Pajdla (ed.), Czech Pattern Recognition Workshop '97*, (Milovy, CZ, 17 - 20 Feb 1997).

Hall, N.S. and S. Laflin, (1984) Computer Aided Design Technique for Pottery Profiles, *Proceeding of Conference Centre for Computing and Computer Science*. Birmingham: 178 - 188.

Huang, Q. X., Flory, S., Gelfand, N., Hofer, M. and Pottmann, H. 2006. Reassembling fractured objects by geometric matching, *ACM Transactions on Graphics*, 25(3): 578.

Kampel, M. and R. Sablatnig (2004) 3-D Puzzling of Archaeological Fragments, DanijelSkocaj (ed.), *Proceedings of 9th Computer Vision Winter Workshop, Piran, Slovenia. Slovenian Pattern Recognition Society*: 31 - 40.

Kampel. M. and R. Sablatnig (2003) Virtual reconstruction of broken and unbroken pottery. *In: Fourth International Conference on 3-D Digital Imaging and Modeling: 3DIM 2003, 6-10 October 2003, Banff, Alberta, Canada: proceedings (Los Alamitos: IEEE Computer Society):* 318 - 325.

Koutsoudis, A., G. Pavlidis, F. Arnaoutoglou, D. Tsiafakis, and C. Chamzas (2009).Qp: A tool for generating 3D models of ancient Greek pottery. *Journal of Cultural Heritage*, *10*(2): 281 - 295.

Lui.Y and H. Pottmann (2006) "Constrained 3d Shape Reconstruction Using a Combination of Surface Fitting and Regiatration", *Computer Aided Design 38:* 572 - 583.

Papaioannou, G., E.A. Karabassi and T. Theoharis, (2002) Reconstruction of three-dimensional objects through matching of their parts," *IEEE Transactions on Pattern Anal.* vol.24, no.1: *IEEE Computer Society Washington, DC, USA:* 114 - 124.

Rasheed, N and Md. Nordin (2015) A Survey of Computer Methods in Reconstruction of 3D Archaeological Pottery Objects, *International Journal of Advanced Research*, 3(3): 712 - 724.

Sablatnig, R. and C. Menard (1997) 3D Reconstruction of archaeological pottery using profile primitives, *Proceeding of the International Workshop on Synthetic-Natural Hybrid Coding and Three-Dimensional Imaging, Rhodes, Greece*: 93 - 96.

Üçoluk, G. and I.H. Toroslu, (1999) "Automatic reconstruction of broken 3-D surfaces objects," *The Science Direct interface, Computer and Graphics*, Vol. 23, no. 4: 573 - 582.

Willis, A. and S. Andrews (2001) Assembling virtual pots from 3D measurements of their fragments, <u>VAST '01</u> Proceedings of the 2001 conference on Virtual reality, archeology, and cultural heritage, North Athens, Greece: ACM Siggraph: 241 - 25.